Common Core State Standards for

Mathematics

Critical Area Learning Descriptions
in Continuum Form

Mathematics Core Standards –Critical Area Learning Descriptions in Continuum Form

Grade K learning

Through learning in **Counting and Cardinality**, **Operations and Algebraic Thinking** and **Number**, and **Operations in Base Ten** students

- Use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Students should see addition and subtraction equations, and student writing of equations is encouraged, but it is not required.)
- Choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

Grade 1 learning

Through learning in **Number and Operations in Base Ten** students

- Develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10.
- Compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes.
- Think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones).
- Understand the order of the counting numbers and their relative magnitudes through activities that build number sense.

Through learning in **Operations and Algebraic Thinking** students

- Develop strategies for adding and subtracting whole numbers based on their prior work with small numbers.
- Use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations.
- Understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two).
- Use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction

Grade 2 learning

Through learning in **Number and Operations in Base Ten** students

- Extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing.
- Understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

Through learning in **Operations and Algebraic Thinking** students

- Use their understanding of addition to develop fluency with addition and subtraction within 100.
- Solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations.
- Select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

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Grade K learning	Grade 1 learning	Grade 2 learning
Through Learning in **Geometry** students • Describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. • identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. • Use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.	Through Learning in **Measurement and Data** students • Develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement (Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term) Through Learning in **Geometry** students • Compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. • Recognize shapes from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry. (through combining shapes)	Through Learning in **Measurement and Data** students • Recognize the need for standard units of measure (centimeter and inch) • Use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. • Recognize that the smaller the unit, the more iterations they need to cover a given length. Through Learning in **Geometry** students • Describe and analyze shapes by examining their sides and angles. • Investigate, describe, and reason about decomposing and combining shapes to make other shapes. • Develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades through building, drawing, and analyzing two- and three-dimensional shapes.

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Grade 3 learning

Through learning in **Operations and Algebraic Thinking**, and **Number and Operations in Base Ten** students

- Develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.
- Use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors.
- Learn the relationship between multiplication and division by comparing a variety of solution strategies,.

Through learning in **Operations and Algebraic Thinking**, and **Number and Operations in Base Ten** students

students

- Develop an understanding of fractions, beginning with unit fractions.
- View fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole.
- Understand that the size of a fractional part is relative to the size of the whole. *For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts.*
- Are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

Grade 4 learning

Through learning in **Operations and Algebraic Thinking**, and **Number and Operations in Base Ten** students

- Generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place.
- Apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers.
- They select and accurately apply appropriate methods to estimate or mentally calculate products, depending on the numbers and the context,
- Develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems.
- Apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends.
- Select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

Through learning in **Number and Operations – Fractions** students

- Develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions.
- Extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

Grade 5 learning

Through learning in **Number and Operations – Fractions**, and **Operations and Algebraic Thinking** students

- Apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators.
- Develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them.
- Use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. *(Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)*

Through learning in **Operations and Algebraic Thinking**, **Number and Operations in Base Ten**, and **Number and Operations – Fractions** students

- Develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations.
- Finalize fluency with multi-digit addition, subtraction, multiplication, and division.
- Apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results.
- Use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense.
- Compute products and quotients of decimals to hundredths efficiently and accurately.

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Grade 3 learning

Through learning in **Measurement and Data** students

- Recognize area as an attribute of two-dimensional regions.
- Measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area.
- Understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

Through learning in **Geometry**, and **Number and Operations – Fractions** students

- Describe, analyze, and compare properties of two dimensional shapes.
- Compare and classify shapes by their sides and angles, and connect these with definitions of shapes.
- Relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 4 learning

Through learning in **Measurement and Data**, and **Geometry** students

- Describe, analyze, compare, and classify two-dimensional shapes.
- Deepen their understanding of properties of two-dimensional objects through building, drawing, and analyzing two-dimensional shapes and using them to solve problems involving symmetry.

Grade 5 learning

Through learning in **Measurement and Data**, and **Geometry** students

- Recognize volume as an attribute of three-dimensional space.
- Understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps.
- Understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume.
- Select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume.
- Decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes.
- Measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

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Grade 6 learning

Through learning in **Ratios and Proportional relationships** students

- Use reasoning about multiplication and division to solve ratio and rate problems about quantities.
- Connect their understanding of multiplication and division with ratios and rates by viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities.
- Expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions.
- Solve a wide variety of problems involving ratios and rates.

Through learning in **The Number System** students

- use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense.
- use operations with rational numbers to solve problems.
- extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers.
- reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

Through learning in **Expressions and Equations** students

- Understand the use of variables in mathematical expressions.
- Write expressions and equations that correspond to given situations,
- Evaluate expressions,
- Use expressions and formulas to solve problems.
- Understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms.
- Know that the solutions of an equation are the values of the variables that make the equation true.
- Use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations.
- Construct and analyze tables, such as tables of quantities that are in equivalent ratios,
- Use equations (such as $3x = y$) to describe relationships between quantities.

Grade 7 learning

Through learning in **Ratios and Proportional relationships** students

- Extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems.
- Use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease.
- Solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects.
- Graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope.
- Distinguish proportional relationships from other relationships.

Through learning in **The Number System**, and **Expressions and Equations** students

- Develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers.
- Extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division.
- Explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers, by applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero),
- Use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

Grade 8 learning

Through learning in **The Number System, Statistics, Expressions and Equations**, and **Ratios and Proportional relationships**

- Use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems.
- Recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin.
- Understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$.
- Use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom).
- Informally fit a model, and assess its fit to data
- Interpret a model in the context of data by expressing a relationship between the two quantities in question and interpret components of the relationship (such as slope and y-intercept) in terms of the situation.
- Strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.
- Solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line.
- Use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

Through learning in **Functions, The Number System, Expressions and Equations,** and **Ratios and Proportional relationships**

- Grasp the concept of a function as a rule that assigns to each input exactly one output.
- Understand that functions describe situations where one quantity determines another.
- Translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they
- Describe how aspects of the function are reflected in the different representations.

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Grade 6 learning

Through learning in **Measurement**, and **Geometry**, students

- Build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume.
- Find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles.
- Discuss, develop, and justify formulas for areas of triangles and parallelograms (using methods above)
- Find areas of polyons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine.
- Reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths.
- Prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Through learning in **Statistics and Probability** students

- Build on and reinforce their understanding of number, beginning to develop their ability to think statistically.
- Recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point.
- Recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability.
- Learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected

Grade 7 learning

Through learning in **Geometry**, students

- Continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three dimensional objects.
- Reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines (in preparation for work on congruence and similarity in Grade 8).
- Work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections.
- Solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

Through learning in **Statistics and Probability** students

- Build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations.
- Begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Grade 8 learning

Through learning in **Geometry** students

- Use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems.
- Show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines.
- Understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways.
- Apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons.
- Solve volume problems involving cones, cylinders, and spheres.